A norm-based point of view for fault diagnosis: Application to aerospace missions.

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Outline of the talk

1. Motivations for "norm-based" theory
2. Objectives and problem setting: the LTI case
3. The solution
4. The μ_g-post-analysis procedure
5. Aerospace examples
6. Concluding remarks (extension to LPV problems)

Preliminaries

matching the keywords "fault detection"

- *The SciVerse - ScienceDirect database*
  → returns more than 40200 records
- *The IEEE database*:
  → returns more than 12970 records
- *The AIAA database*:
  → returns more than 200 records

- FDI: a very active research topic in both academic and aerospace industrial institutions
- Not an exhaustive list of existing methods (hardware and model-based, FDI / DX communities)
- Model-based methods applied (with potential applications) to aerospace problems

Motivations

- Two main approaches
  - Fault estimation ⇒ min optimization problem e.g. following the Kalman's *"Prediction - Correction cycle"* or the *"Unknown Input estimation technique"*.
  - Residual generation problem ⇒ min / max optimization problem

Linear approaches

$$r(s) = H_y(s)y(s) + H_u(s)u(s) \quad r(s) = G_d(\theta, s)d(s) + G_f(\theta, s)f(s)$$

- Two main solutions

Decoupling approaches

$G_d(\theta, s)d(s) = 0$
$G_f(\theta, s)f(s) \Rightarrow$ fault detectability.

- "perfect" robustness level
- $\perp$ *span*$\{d\}$ and // *span*$\{f\}$ may not exist
- Isolation problem: Rank conditions (often) not satisfied

An alternative framework: "Approximated" decoupling

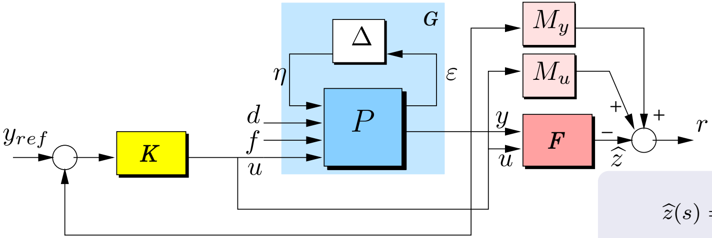
$$\Rightarrow \min h_1(T_{rd}) \,/\, \max h_2(T_{rf}) \text{ with poles assignment}$$

$$h_1 \in \{H_2, H_{2g}, H_\infty\} \quad h_2 \in \{H_-, H(0)\}$$

1. H_∞ specifications: to enforce robustness to model uncertainty (e.g. external disturbances, parametric uncertainties and neglected dynamics).
2. $H(0)/H_-$ specifications: for fault sensitivity requirements over specified frequency ranges.
3. H_2 objectives: to take into account the stochastic nature of disturbances.
4. H_{2g} + poles assignment: to tune the transient response and to enforce some minimum decay rate of the residuals.

Remark: *H_2 not useful for fault sensitivity specifications (Zhou,2009).*

The proposed solution



$$\hat{z}(s) = F(s)\begin{pmatrix} y(s) \\ u(s) \end{pmatrix}$$

$$r(t) = M_y y(t) + M_u u(t) - \hat{z}(t)$$

- The LFR paradigm: nonlinear parametric uncertainties, neglected dynamics, flexible modes (Beck et al., 1996; Cockburn and Morton, 1997; Varga et al., 1998; Beck and Doyle, 1999; Hecker et al., 2005)

$$y = \mathcal{F}_u(P, \Delta)\begin{pmatrix} d^T & f^T & u^T \end{pmatrix}^T, \Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1,$$
$$\underline{\Delta} = \{\text{block diag}(\delta_1^r I_{k_1}, ..., \delta_{m_r}^r I_{k_{m_r}}, \delta_1^c I_{k_{m_r+1}}, ..., \delta_{m_c}^c I_{k_{m_r+m_c}}, \Delta_1^C, ..., \Delta_{m_C}^C), \delta_i^r \in \mathbb{R}, \delta_i^c \in \mathbb{C}, \Delta_i^C \in \mathbb{C}\}$$

- Takes into account the controller actions
- M_y, M_u: "merge optimally" input/output signals

Problem formulation

Let f be detectable faults (Massoumnia et al., 1989; Chung and Speyer, 1998; Zad and Massoumnia, 1999; Saberi et al., 2000) and assume $\sup_\omega \mu_{\underline{\Delta}} \mathcal{F}_l\left((P,K)(j\omega)\right) < 1$. The goal is to find $M_y \in \mathbb{R}^{q_r \times m}$, $M_u \in \mathbb{R}^{q_r \times p}$ (constant), $A_F \in \mathbb{R}^{n_F \times n_F}$, $B_F \in \mathbb{R}^{n_F \times (p+m)}$, $C_F \in \mathbb{R}^{q_r \times n_F}$ and $D_F \in \mathbb{R}^{q_r \times (p+m)}$ so that r solves the optimization problem:

1. $\min_{M_y, M_u, F} \gamma_1 \quad \begin{array}{l} \forall \Delta \in \underline{\Delta} : \|\Delta\|_\infty \leq 1 \\ \text{s.t. } \|T_{rd}\|_\infty < \gamma_1 \end{array}$ (Robustness)
2. $\max_{M_y, M_u, F} \gamma_2 \quad \begin{array}{l} \forall \Delta \in \underline{\Delta} : \|\Delta\|_\infty \leq 1 \\ \text{s.t. } \|T_{rf}\|_- > \gamma_2 \quad \forall \omega \in \Omega \end{array}$ (Fault sensitivity)
3. $\lambda_i \{A_F\} \in \mathcal{D}$ with $\mathcal{D}$ left-half plane (Poles assignment)

Remark: The smallest gain (Chen & Patton,1999; Rank & Niemann,1999; Henry, 2005): $\|P\|_- = \inf_{\omega \in \Omega} \underline{\sigma}(P(j\omega)), \Omega = [\omega_1 \,;\, \omega_2]$
→ differs from (Liu et al., 2005; Wei and Verhaegen, 2008), i.e. defined on a infinite frequency horizon

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The LMI solution ..... derived in 3 steps

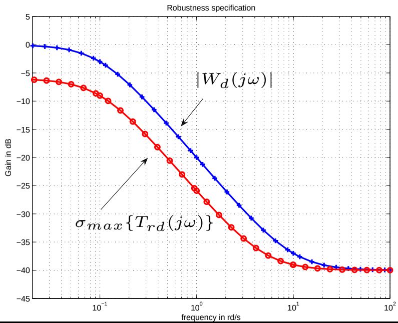
1. Specify the robustness and sensitivity objectives + poles assignment → "shaping filters" + LMI Regions.
2. The quasi-standard form → algebra manipulations on LFRs.
3. Use of the bounded real (Boyd, 1994) and the projection lemmas (Gahinet & Apkarian, 1994) using an appropriate basis, to derive a SDP formulation.

Robustness objectives (shaping filter W_d)

Let $W_d : ||W_d||_\infty \leq \gamma_1$ be the shaping filters associated to T_{rd}. Then the robustness requirements are satisfied iff

$$\exists M_y, M_y, F : ||T_{rd} W_d^{-1}||_\infty < 1 \quad \square$$

- Illustrative example: noise measurement rejection (to understand).
- Objective: attenuation of $40dB$ (**at least**) for $\omega \in [10rd/s; +\infty[$



$$W_d(s) = 10^{-2} \frac{s+10}{s+0.1}$$

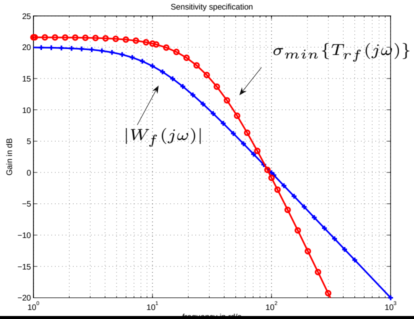
$$\sigma_{max}\{T_{rd}(j\omega)\} < |W_d(j\omega)|$$
$$\Leftrightarrow ||T_{rd} W_d^{-1}||_\infty < 1$$

Fault sensitivity objectives (shaping filter W_f)

Let W_F be an invertible LTI transfer matrix defined such that $||W_f||_- = \gamma_2/\lambda ||W_F||_-$ and $||W_F||_- > \lambda$ where $\lambda = 1 + \gamma_2$. Define the (fictitious) signal $\tilde{r}$ such that $\tilde{r} = r - W_F f$. Then fault sensitivity objective is satisfied if (sufficient condition)

$$\exists M_y, M_u, F : \|T_{\tilde{r}f}\|_\infty < 1 \quad \square$$

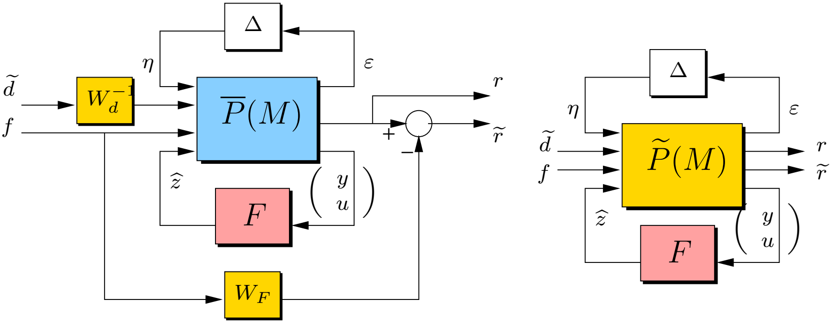
Proof: (Henry, 2005)



Illustrative example
$\Rightarrow$ amplification of $20dB$ (**at least**) for $\omega \in]0; 100]rd/s$

$$\sigma_{min}\{T_{rf}(j\omega)\} > |W_f(j\omega)| \Leftrightarrow ||T_{rf}W_f^{-1}||_- > 1 \forall \omega \in \Omega$$

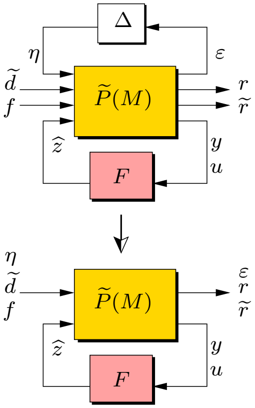
The quasi-standard problem



Let $M = [M_y \ M_u] \in \mathbb{R}^{q_r \times (m+p)}$. Including W_d^{-1} and W_F into $\overline{P}(M)$. Then, after some LFR manipulations, it follows that $(M_y, M_u, F(s))$ solves the problem iff:

$$\left\| \mathcal{F}_u \left(\mathcal{F}_l \left(\widetilde{P}(M), F \right), \Delta \right) \right\|_\infty < 1$$

The quasi-standard problem



Small gain theorem (Zames,64)

$$\left\| \mathcal{F}_u \left(\mathcal{F}_l \left(\tilde{P}(M), F \right), \Delta \right) \right\|_\infty < 1 \quad \forall \Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$$

if $\left\| \mathcal{F}_l \left(\tilde{P}(M), F \right) \right\|_\infty < 1$ □

$$\tilde{P}(M) = \left[\begin{array}{c|cc} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \tilde{D}_{22} \end{array} \right]$$

$$\tilde{B}_2 = 0, \tilde{D}_{22} = 0, \tilde{D}_{12} = \left(\begin{array}{c} 0 \\ -I_{2\times q_r} \end{array} \right)$$

Remark: Not a standard H_∞ problem since $\tilde{P}(M)$ $\quad M = (M_y \; M_u)$
⇒ *"hinfsyn" / "hinflmi" not applicable*

The SDP formulation

Proposition (Henry, 2005)

Let $W = \begin{pmatrix} \widetilde{C}_2 & \widetilde{D}_{21} \end{pmatrix}^{\perp}$. Then $(M_y, M_u, F(s))$ solve the SDP problem **iff** $\exists M_y \in \mathbb{R}^{q_r \times m}$, $M_u \in \mathbb{R}^{q_r \times p}$ and matrices $R = R^T > 0$ and $S = S^T > 0$ that solve the following SDP problem:

$$
\begin{aligned}
&\min \gamma \\
&s.t. \begin{pmatrix} \widetilde{A}R + R\widetilde{A}^T & R\widehat{C}_1^T & \widetilde{B}_1 \\ \widehat{C}_1 R & -\gamma I_w & \widehat{D}_{11} \\ \widetilde{B}_1^T & \widehat{D}_{11}^T & -\gamma I_{v+q_{\tilde{d}}+q_f} \end{pmatrix} < 0 \\
&\left(\begin{array}{c|c} W & 0 \\ \hline 0 & I \end{array}\right)^T \left(\begin{array}{cc|c} \widetilde{A}^T S + S\widetilde{A} & S\widetilde{B}_1 & \widetilde{C}_1^T(M) \\ \widetilde{B}_1^T S & -\gamma I_{v+q_{\tilde{d}}+q_f} & \widetilde{D}_{11}^T(M) \\ \hline \widetilde{C}_1(M) & \widetilde{D}_{11}(M) & -\gamma I_{w+2q_r} \end{array}\right) \left(\begin{array}{c|c} W & 0 \\ \hline 0 & I \end{array}\right) < 0 \\
&\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0
\end{aligned}
$$

$$
\widehat{C}_1 = \begin{pmatrix} C_1 & D_{1d}C_{wd} & 0_{w \times n_{wF}} \end{pmatrix}, \quad \widehat{D}_{11} = \begin{pmatrix} D_{11} & D_{1d}D_{wd} & D_{1f} \end{pmatrix}
$$

Proof Bounded real lemma + projection lemma in a judiciously chosen basis. □

The numerical procedure

1. Solve the SDP problem (SDPT3, CSDP, DSDP5 ...etc...) $\Longrightarrow$ the optimal solution (γ, R, S, M_y, M_u)
2. Compute M, N so that $MN^T = I - RS \rightarrow$ SVD
3. Compute the Lyapunov matrix X_{cl} (those involves in the bounded real lemma)

$$\begin{pmatrix} R & I \\ M^T & 0 \end{pmatrix} = \begin{pmatrix} I & S \\ 0 & N^T \end{pmatrix}$$

4. Find A_F, B_F, C_F, D_F so that

$$\begin{pmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cl} & -\gamma I_{v+q_{\tilde{d}}+q_f} & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I_{w+2q_r} \end{pmatrix} < 0$$

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$\rightarrow$ using a SDP solver (SDPT3, CSDP, DSDP5 ...etc...)
$\rightarrow$ using some linear algebra (Gahinet et Apkarian, 94)

Computational issues

- Since the procedure involves $MN^T = I - RS \Rightarrow I - RS$ has to be well conditioned .
- Unfortunately, $I - RS$ will be nearly singular if the constraint $\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0$ is satured at the optimum.

Solution

Maximize the minimal eigenvalue of RS to push all eigenvalues away from "1", i.e.

$$\min \gamma + \varepsilon\rho \quad s.t$$
$$\begin{pmatrix} R & \rho I \\ \rho I & S \end{pmatrix} \geq 0 \qquad \text{with } \varepsilon < 0$$

Computational issues

Since we optimize M_y, M_u and F simultaneously
$\Rightarrow$ no unique solution for M_y, M_u and D_F, i.e. the static part of F
$\rightarrow r(t)$ includes the subtraction of these two parts.

Solution

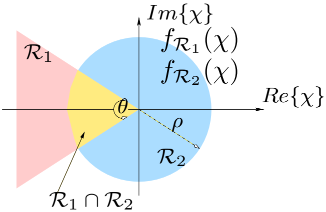
1. Add constraints on $M = [M_y \ M_u]$ to the LMIs
2. and/or add an optimization objective on some functional of M

$$\sum_j M_{ij} = 1, \ \forall i$$

Remark: *Following my own experience:* $D_F = 0$ *(Henry,2005; Henry,2006; ,Falcoz et al. 2008; Henry, 2008; Falcoz et al., 2010).*

Robust poles assignment

<u>Definition</u>(Chilali et Gahinet, 1996):
$\mathcal{R}$ is called a LMI region if $\exists\ L = L^T, Q = Q^T$:
$\mathcal{R} = \{\chi \in \mathbb{C} : f_{\mathcal{R}}(\chi) = L + \chi Q + \chi^* Q^T < 0\}$
$f_{\mathcal{R}}(\chi)$ = characteristic function of $\mathcal{R}$.



Proposition (Henry, 2005b)

$\lambda_i\{A_F\} \in \mathcal{R} = \mathcal{R}_1 \cap \ldots \cap \mathcal{R}_N,\ \forall i,\ \forall \Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$ if (sufficient condition) $\exists N\ X_i = X_i^T > 0$ t.q.

$$\begin{pmatrix} \mathcal{Q}_{\mathcal{R}_i}(A_{\mathcal{R}}, X_i) & Q_{1i}^T \otimes X_i B_{\mathcal{R}} & Q_{2i}^T \otimes C_{\mathcal{R}}^T \\ Q_{1i} \otimes B_{\mathcal{R}}^T X_i & -I & I \otimes D_{\mathcal{R}}^T \\ Q_{2i} \otimes C_{\mathcal{R}} & I \otimes D_{\mathcal{R}} & -I \end{pmatrix} < 0 \quad i = 1, \ldots, N$$

$\rightarrow \mathcal{Q}_{\mathcal{R}_i}(A_{\mathcal{R}}, X_i) = L_i \otimes X_i + Q_i \otimes X_i A_{\mathcal{R}} + Q_i^T \otimes A_{\mathcal{R}}^T X_i$
$\rightarrow A_{\mathcal{R}}, B_{\mathcal{R}}, C_{\mathcal{R}}, D_{\mathcal{R}}$: transfer $(\eta^T\ \hat{z}^T)^T \rightarrow (\varepsilon^T\ y^T\ u^T)^T$ □

H_{2g} specifications

Goal: Find $(M_y, M_u, F(s))$ so that.

$$|r(t)|_{||d||_2=1} < \gamma_3 \quad \forall t \Leftrightarrow ||T_{rd}||_{2g} < \gamma_3$$

Proposition (Henry, 2005b)

Given a fictitious signal $r_g \in \mathbb{R}^{q_r}$ so that $r_g = W_g r \quad W_g \in \mathbb{R}^{q_r \times q_r}$, $||W_g|| = 1/\gamma_3$ and $(A_g, B_g, C_g, D_g) \Rightarrow (d^T\ \hat{z}^T)^T \longrightarrow (r^T\ y^T\ u^T)^T$. Then $||T_{rd}||_{2g} < \gamma_3$ if (necessary condition) $\exists X_g = X_g^T > 0$ and $\alpha < 1$ so that:

$$\begin{aligned} &\min \alpha \\ &s.t. \quad \begin{pmatrix} A_g^T X_g + X_g A_g & X_g B_g \\ B_g^T X_g & -I \end{pmatrix} < 0, \quad \begin{pmatrix} X_g & C_g^T \\ C_g & \alpha I \end{pmatrix} > 0 \\ &\qquad D_g = 0 \end{aligned}$$

Proof: Based on some results in (Scherer et al, 1997) □

Computational procedure

Join all LMIs into a single optimization problem, i.e. find A_F, B_F, C_F, D_F so that:

- H_∞/H_- :
$$\begin{pmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cl} & -\gamma I_{v+q_{\tilde{d}}+q_f} & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I_{w+2q_r} \end{pmatrix} < 0$$

- Robust poles assignment:
$$\begin{pmatrix} \mathcal{Q}_{\mathcal{R}_i}(A_{\mathcal{R}}, X_i) & Q_{1i}^T \otimes X_i B_{\mathcal{R}} & Q_{2i}^T \otimes C_{\mathcal{R}}^T \\ Q_{1i} \otimes B_{\mathcal{R}}^T X_i & -I & I \otimes D_{\mathcal{R}}^T \\ Q_{2i} \otimes C_{\mathcal{R}} & I \otimes D_{\mathcal{R}} & -I \end{pmatrix} < 0 \quad i = 1, ..., N$$

- H_{2g} specifications:
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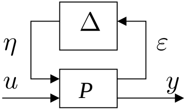
For convexity reasons: $X_{cl} = X_i = X_g$
$\Rightarrow$ conservative solutions.

An illustrative example

$$\begin{cases} \dot{x} = A(k)x + E_1 d + K_1 f \\ y = Cx \end{cases}, A(k) = \begin{pmatrix} 0 & 1 \\ -k/m & -\zeta/m \end{pmatrix}, C = I_2$$

$$m = 10, \quad \zeta = 10, \quad 8 < k < 12, \quad E_1 = (1\ 1)^T, \quad K_1 = (1\ 1)^T$$

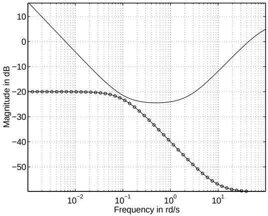
- Step 1: The LFR formulation:



$$y = \mathcal{F}_u(P, \Delta)$$
$$\Delta = \delta_k, \delta_k \in \mathbb{R}, |\delta_k| \leq 1$$
$$k = k_0 + w_k \delta_k,\ k_0 = 10,\ w_k = 2.$$

- Step 2: The shaping filters:

→ f in low frequencies
→ d in a limited frequency range



- Step 3: Solve the SDP problem and compute $F(s)$:

```
Probleme de Synthese de F sur les dimensions:
--> Nb d:  1 / Nb f:  1/ Nb u:1/ Nb y:  1/ Nb r:  1
+ Probleme d'optimization LMI:
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
  ID       Constraint                        Type        Tag
   1    Numeric value      Matrix inequality 8x8     LMI n1
   2    Numeric value      Matrix inequality 8x8     LMI n2
   3    Numeric value    Matrix inequality 12x12     LMI RS
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
+ Minimisation de gamma
+ Solver chosen :  sdpt3-3.02
+ Processing objective h(x)
+ Processing F(x)
+ Calling SDPT3
***************************************************************
Infeasible path-following algorithms
***************************************************************

  version    predcorr      gam    expon    scale data
       NT           1    0.000        1             0
```

it	pstep	dstep	p infeas	d infeas	gap	obj	cputime
0	0.000	0.000	9.7e+05	1.0e+01	2.7e+06	-1.470782e-06	
1	0.002	0.033	9.7e+05	9.7e+00	2.7e+06	-2.005763e+02	0.3
2	0.041	0.061	9.3e+05	9.1e+00	2.7e+06	-1.188450e+03	0.3
3	0.174	0.782	7.7e+05	2.0e+00	1.8e+06	-8.102019e+03	0.3
4	0.694	0.844	2.4e+05	3.1e-01	6.7e+05	-1.143302e+04	0.3
5	0.746	0.623	6.0e+04	1.2e-01	2.3e+05	-1.089181e+04	0.3
6	0.878	0.896	7.3e+03	1.2e-02	3.4e+04	-6.294910e+03	0.3
7	0.626	0.800	2.7e+03	2.4e-03	1.6e+04	-3.533212e+03	0.4
8	0.402	1.000	1.6e+03	1.3e-14	1.1e+04	-1.555054e+03	0.4
9	0.850	0.926	2.4e+02	5.2e-15	1.9e+03	-6.850118e+02	0.4

19	0.798	0.565	1.4e-04	8.2e-13	8.5e-01	-9.484335e-01	0.5
20	0.678	1.000	4.5e-05	9.2e-13	5.5e-01	-9.811465e-01	0.6
21	0.966	0.955	1.5e-06	8.2e-13	2.5e-02	-9.857646e-01	0.6
22	0.907	0.967	1.4e-07	2.5e-12	2.4e-03	-9.854703e-01	0.6
23	0.935	1.000	1.1e-08	1.1e-12	9.3e-04	-9.854826e-01	0.6
24	0.940	0.964	3.0e-09	1.6e-12	5.4e-05	-9.855364e-01	0.6
25	1.000	0.899	1.8e-08	4.6e-12	1.6e-05	-9.855383e-01	0.6
26	1.000	1.000	1.0e-08	3.0e-12	6.9e-07	-9.855386e-01	0.6
27	1.000	1.000	1.1e-08	3.4e-12	1.5e-08	-9.855386e-01	0.7

```
Stop:  max(relative gap, infeasibilities) < 1.00e-07
----------------------------------------------------
  number of iterations       = 27
  primal objective value     = -9.85553456e-01
  dual objective value       = -9.85523662e-01
  actual relative gap        = -1.50e-05
  gap := trace(XZ)           = 1.51e-08
  relative gap               = 1.51e-08
  primal infeasibilities     = 1.06e-08
  dual infeasibilities       = 3.41e-12
  Total CPU time (secs)      = 0.8
  CPU time per iteration     = 0.0
  termination code           = 0                     info:  'No problems detected (SDPT3)'
----------------------------------------------------
> gamma minimal:  0.98552
> M=[-0.6285 -4.8167e+03]          DF=[0 0]
+ Processing Ptilde
+ Processing myklmi => rank(I-RS)< n => dim(Aaug):6 - dim(AF):5
+ Processing postanalyse
Performance Hinf obtenue:  De klmi: 0.98552 - De norminf: 0.98786
```

All numerical indicators are green !

The μ_g analysis procedure

- The procedure involves sufficient conditions ($H_- \rightarrow H_\infty$, small gain theorem)

- The nature (i.e. real and/or complex) and the structure (block diagonal) of Δ is not taken into account.

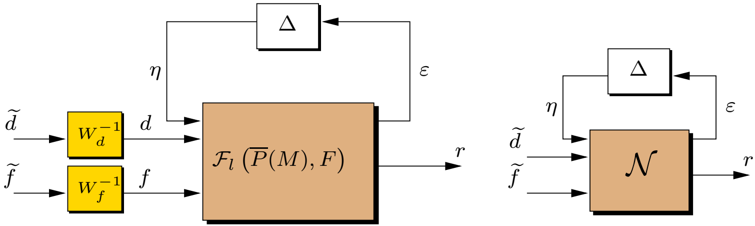
$\Rightarrow \gamma < 1 \rightarrow$ what about the conservativeness ?
$\Rightarrow \gamma \geq 1 \rightarrow F(s), M_y, M_u =$ May be an admissible solution !

Post-analysis of robust fault detection performance

$\Rightarrow$ the generalized structured singular value μ_g (Newlin and Smith,1998; Henry 2002;Henry,2005; Henry,2006; Falcoz et al. 2010)

The μ_g analysis procedure

Consider W_d and W_f. With some LFR manipulations ....



Problem formulation

With the computed solution $M_y, M_u, F(s)$:

$$||T_{r\tilde{d}}||_\infty < 1 \text{ and } ||T_{r\tilde{f}}||_- > 1 \quad \forall\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$$

or (which is strictly equivalent)

$$\sup_\omega \overline{\sigma}\left(T_{r\tilde{d}}(j\omega)\right) < 1 \text{ and } \inf_{\omega\in\Omega} \underline{\sigma}\left(T_{r\tilde{f}}(j\omega)\right) > 1 \quad \forall\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$$

The μ_g analysis procedure

Theorem (Henry, 2005)

Let $\mathcal{N} = \begin{pmatrix} \mathcal{N}_{11} & \mathcal{N}_{12} \\ \mathcal{N}_{21} & \mathcal{N}_{22} \end{pmatrix}$ so that $\mathcal{N}_{22} = T_{r\tilde{f}}|_{\Delta=0}$. Assume that

$$\sup_{\omega} \mu_{\underline{\overline{\Delta}}}(\mathcal{N}_{11}(j\omega)) < 1$$

where $\underline{\overline{\Delta}} = \{diag(\Delta, \Delta_d)\}$. $\Delta_d \in \mathbb{C}^{q_{\tilde{d}} \times q_r}$. Assume that $\mathcal{N} \in \mathrm{dom}(\mu_g)$. Then a necessary and sufficient condition for $(M_y, M_u, F(s))$ to satisfy the requirements W_d/W_f is:

$$\boxed{\sup_{\omega\in\Omega} \mu_{g\underline{\widehat{\Delta}}}(\mathcal{N}(j\omega)) < 1}$$

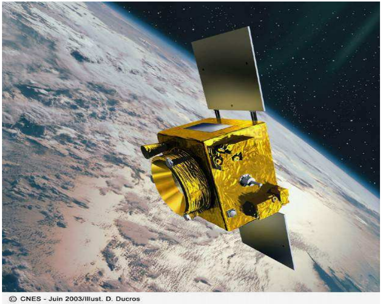
$\underline{\widehat{\Delta}} = \{\mathrm{diag}(\overline{\Delta}, \Delta_f)\}$. $\overline{\Delta}$ is the structure associated to $\underline{\Delta}$ and $\Delta_f \in \mathbb{C}^{q_{\tilde{f}} \times q_r}$.

Applications:

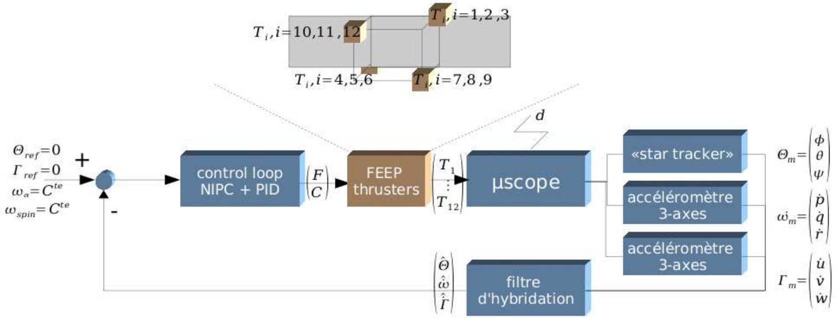
1. The *Microscope* satellite - CNES / IMS (Henry,2008)
2. The *HL20* Reentry Vehicle - ESA / Astrium ST / IMS (Falcoz et al., 2008 - 2010)
3. The *MSR* Rendez-vous mission - Thales Alenia Space / IMS

Microscope

- Scheduled to be launched in 2011
- Mission: test the validity of the equivalence principle (Einstein, 1911) using space-based measurements with a precision higher than 10^{-13}.
- Orbit: circular, quasi polar, sun synchronous - Alt: 700km
- Disturbances
  - Photonic pressure: solar / albedo terrestrial
  - Earth magnetic field
  - Earth gravitational field
  - High atmospheric winds
  - Measurement noises (non perfect NAV module)
- Fault to be detected
  - Actuator (ionic thrusters) faults



Microscope



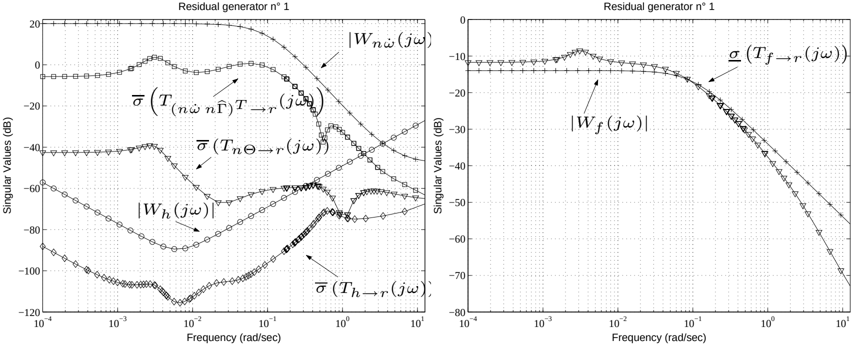
Taking into account the AOCS actions (PID + allocation module) + actuator models + satellite dynamics (small angles assumption) + NAV (Pade approximation + former filters) + additive faults model

$$y = P \begin{pmatrix} d \\ f \\ u \end{pmatrix}, \; u = Ky \quad (i.e.\ \Delta = 0)$$

12 H_∞/H_- filters (DOS strategy)

Microscope

- The shaping filters (and the principal gains of the transfers)

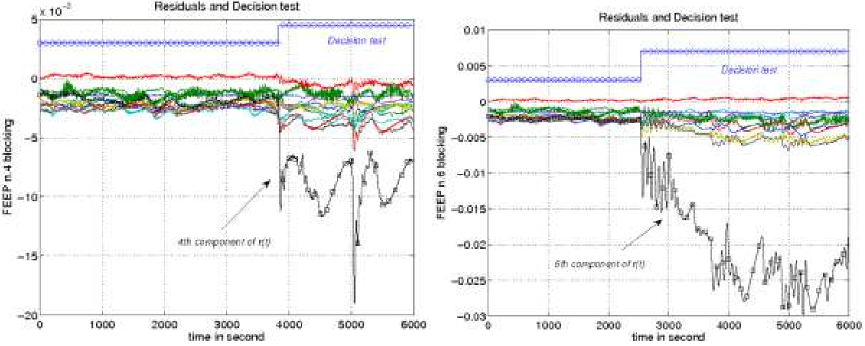


Robustness requirements (left) and Fault sensitivity objectives (right)

- $W_h \rightarrow$ centered around ω_{spin}
- $W_n \rightarrow$ Star Tracker (constant) - Accelerometers (high frq)
- $W_f \rightarrow$ Low frequency requirements $\omega \in]0; 0.1] rd/s$

Microscope

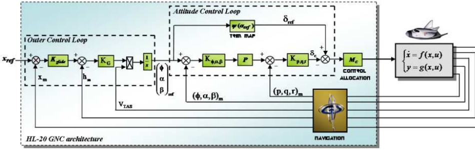
- Nonlinear simulations



Thruster n.4 closed (left) - thruster n.6 jammed (right)

The HL-20 RLV

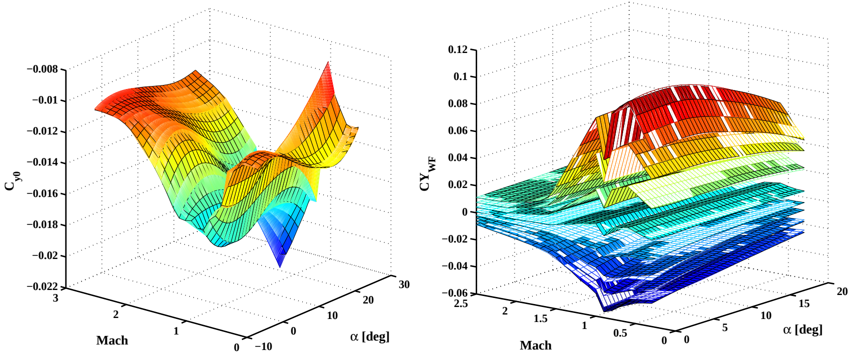
- Flight trajectory: TAEM (mach 2.5 → ma 0.5) + Auto-landing (mach 0.5 → runway)
- Model perturbations:
CoG, mass, inertia, aeros. coeff
- Disturbances:
- Winds and Atmospheric disturbances
- Non perfect NAV
- Faults to be diagnosed: Left/Right flaps
- Modeling the GNC
- gain-scheduling control laws
- dynamic pressure dependent allocation module
- The HL-20 dynamics
- trim analysis (fault free / faulty situations)
- linear approximation around the flight trajectory



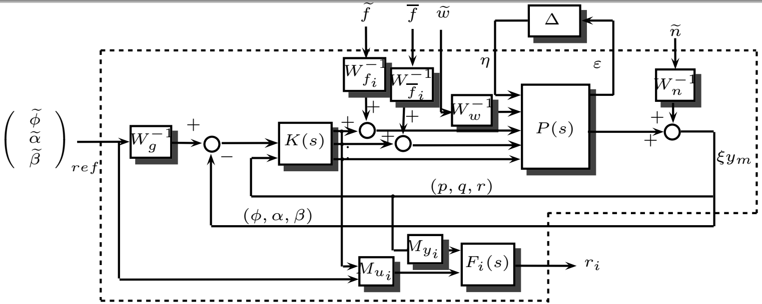
2 H_∞/H_- filters (DOS strategy)

The HL-20 RLV

Modeling the aerodynamic coefficients:
→ Polynomial interpolation (1D) + PCA (2D): auto-landing phase
→ $\mathcal{C}^\infty$ neural network-based models: TAEM phase



$C_{y0}(\alpha, mach)$ (lateral lift) and $C_{Y_{wfl}}(\alpha, \beta, mach)$ (left wing flap)



$$\Delta = diag\{\delta_{C_{l0}}, \delta_{C_{m0}}, \delta_{C_{n0}}, \delta_{C_{x0}}, \delta_{C_{y0}} I_3, \delta_{C_{z0}} I_3, \delta_{X_{cg}} I_2, \delta_{I_{xx}} I_2, \delta_{I_{yy}} I_3, \delta_{I_{zz}} I_3, \delta_m I_3\}$$
$$\Rightarrow \Delta \in \mathbb{R},\ |\delta_i| \leq 1$$

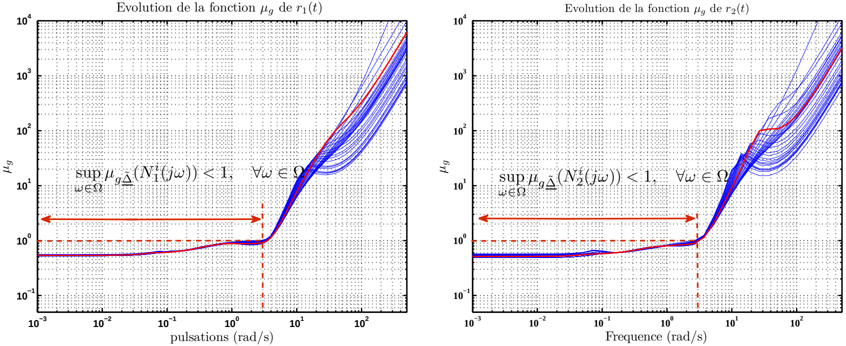
- Atmospheric disturbances (Dryden filters): $W_{wu} = \gamma_w (\sigma_{ug} \sqrt{\frac{L_{ug}}{\pi V}} \frac{1+\tau s}{1+\frac{L_{ug}}{V_{TAS}} s})^{-1}$

$$W_{wv} = W_{ww} = \gamma_w (\sigma_{vg} \sqrt{\frac{L_{vg}}{\pi V}} \frac{1+\frac{\sqrt{3} L_v}{V} s + \tau s}{(1+\frac{L_{vg}}{V} s)^2})^{-1}$$

- Guidance signals: $W_g = \gamma_g I_3$
- Navigation module: $W_n = \gamma_n \frac{1+\frac{1}{250}s}{1+\frac{1}{100}s} . I_{10}$
- Fault sensitivity objectives: $W_{f_i} = \gamma_{f_i} \frac{1}{1+\tau_{f_i} s}$, $\tau_{f_i} = \frac{1}{\omega_{f_i}}$, $\omega_{f_i} = 3rd/s$, $i = 1, 2$

The HL-20 RLV

- The μ_g-analysis (V_{ref} parametrization)

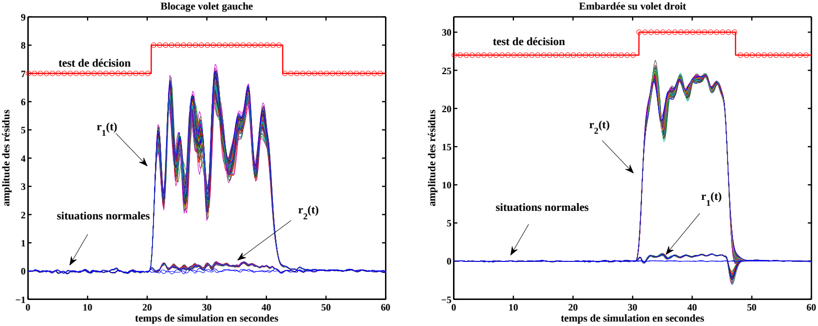


$$\sup_{\omega\in\Omega} \mu_{g\hat{\underline{\Delta}}}(\mathcal{N}_i(j\omega)) < 1,\ i=1,2,\ \Omega=[0;3rd/s]$$

All required robustness/fault sensitivity specifications are met $\forall\Delta$, all along the reentry flight trajectory.

The HL-20 RLV

- Nonlinear simulations (Monte-Carlo campaign)



Left wing flap jammed (left) - right wing flap runaway (right)

The MSR rendez-vous mission

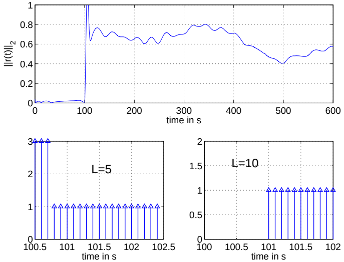
- Mission: Return tangible samples from Mars atmosphere/ground.

- Five spacecrafts: an Earth/Mars transfer vehicle, an orbiter, a descent module, an ascent module and an Earth re-entry vehicle ⇒ Focus on the RDV phase

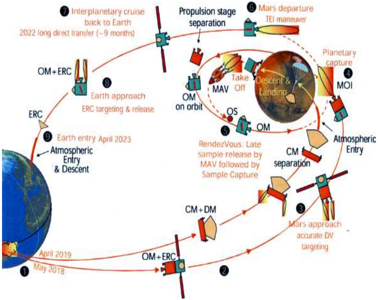
- Fault to be detected: Thruster faults

- FDI scheme:

- A $H(0)$ observer + $\lambda_i\{A - LC\} < \Lambda_i$
- Cross-correlation test:
$\min_i corr\{r_j(t), u_{thr_i}(t+\tau)\}\ i = 1...8$



Fault in thruster 1



Concluding remarks: Extension to LPV problems

LPV formalism = an efficient paradigm to model nonlinear systems

- qLPV formulation (Biannic,1996; Leith,2000; Bruzelius,2004; Marcos,2005...)

$$\begin{cases} \dot{x}_1 = -x_2 \\ \dot{x}_2 = x_1 + \alpha(1 - x_1^2)x_2 \end{cases} \overset{\theta = x_1}{\Longrightarrow} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & \alpha(1-\theta^2) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

- First order approximation around a reference trajectory

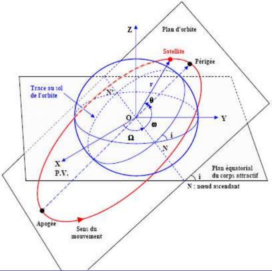
⇒ an adequate formalism for aerospace missions

⇒ known trajectory (x_0, u_0)

$$\begin{cases} \dot{x} = f(x,u) \\ y = g(x,u) \end{cases} \overset{\theta = [x_0^T \ u_0^T]^T}{\Longrightarrow} \begin{cases} \delta\dot{x} = A(\theta)\delta x + B(\theta)\delta \\ \delta y = C(\theta)\delta x + D(\theta)\delta \end{cases}$$

$$A(\theta) = \nabla_x f(x_0, u_0), \quad B(\theta) = \nabla_u f(x_0, u_0)$$

$$C(\theta) = \nabla_x g(x_0, u_0), \quad D(\theta) = \nabla_u g(x_0, u_0)$$



Concluding remarks: Extension to LPV problems

- A framework to model gain-scheduling controllers, NDI-based control laws...(Papageorgiou,2005; Falcoz,2007;Bokor,2010)

Performance measures: An induced I/O signal-based definition

$$||T_{rd}(\theta)||_\infty = \sup_{\substack{\forall \theta \\ ||d||_2 \neq 0}} \frac{||r||_2}{||d||_2} \quad ||T_{rf}(\theta)||_- = \inf_{\substack{\forall \theta \\ ||f||_e \neq 0}} \frac{||r||_e}{||f||_e}, dim(r) = 1$$

robustness

fault sensitivity
(Grenaille & Henry, 2008; Henry, 2009)

Concluding remarks: Extension to LPV problems

- A framework to model gain-scheduling controllers, NDI-based control laws...(Papageorgiou,2005; Falcoz,2007;Bokor,2010)

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robustness

fault sensitivity
(Grenaille & Henry, 2008; Henry, 2009)

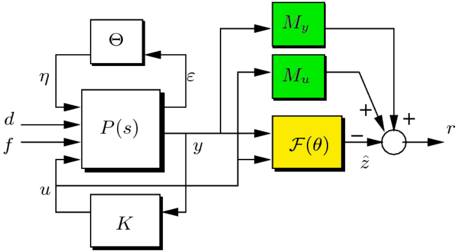
Concluding remarks: Extension to LPV problems

Assumptions: $\theta_i(t)$ are measured in real time and bounded ,i.e. $|\theta_i(t)| \leq 1 \forall t$

$$y = F_u\left(P, \Theta\right) \begin{pmatrix} d \\ f_i \\ u \end{pmatrix}, \quad \begin{array}{l} d = \left(w^T \; v^T \; \overline{f}_i^T\right)^T \\ y = Ku \end{array}$$

$$\Theta = blockdiag\left(\theta_1 I_{k_1}, ..., \theta_q I_{k_q}\right) \text{ so that}$$

$$\Theta \subset \mathbb{R}, \quad ||\Theta||_\infty \leq 1$$



Problem:

Derive M_y, M_u and the (stable) LPV filter $\mathcal{F}(\theta) = F_l(F, \Theta)$ so that

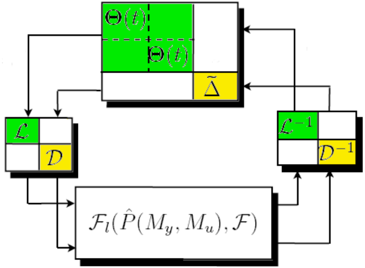
$$F(s) = \begin{pmatrix} C_{F1} \\ C_{F\theta} \end{pmatrix} (sI - A_F)^{-1} \left(B_{F1} \; B_{F\theta}\right) + \begin{pmatrix} D_{F11} & D_{F1\theta} \\ D_{F\theta 1} & D_{F\theta\theta} \end{pmatrix} \text{ that solve}$$

the following optimization problem:

$$\min_{M_y, M_u, F} \gamma_1 \quad s.t. \quad ||T_{d \to r}(\theta)||_\infty < \gamma_1$$

$$\max_{M_y, M_u, F} \gamma_2 \quad s.t. \quad ||T_{f \to r}(\theta)||_{sens} > \gamma_2$$

- *Solution: the scaling matrices technique (Henry,2009)*



Let L_Δ and $L_\mathcal{D}$ be the sets of matrices:

$$L_\Delta = \{L > 0 : L\Theta = \Theta L, \forall \Theta \in \Delta\}$$
$$\Rightarrow L_{\Delta\oplus\Delta} = \{\mathcal{L}diag(\Theta,\Theta) = diag(\Theta,\Theta)\mathcal{L}\}$$
$$L_\mathcal{D} = \left\{\mathcal{D} > 0 : \mathcal{D}\widetilde{\Delta} = \widetilde{\Delta}\mathcal{D},\ \forall\widetilde{\Delta}\right\}$$

The scaled H_∞ problem

$\mathcal{F}(\theta)$ is internally stable $\forall\theta(t)$ and there exists a solution to the problem if $\exists\gamma < 1, M_y, M_u$ and $F(s)$:

$$\left\| \begin{pmatrix} \mathcal{L}^{1/2} & 0 \\ 0 & \frac{1}{\sqrt{\gamma}}\mathcal{D} \end{pmatrix} F_l\left(\widetilde{P}(M_y, M_u), F\right) \begin{pmatrix} \mathcal{L}^{-1/2} & 0 \\ 0 & \frac{1}{\sqrt{\gamma}}\mathcal{D}^{-1} \end{pmatrix} \right\|_\infty < 1$$

Pictures from space: ATV de-docking

1. Docking: 3rd April 2008
2. De-docking: 5th September 2008

Thank you for listening