Smith Predictor Based Control of Continuous-Review Perishable Inventory Systems with a Single Supply Source

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Abstract: In this paper we address the problem of efficient control of continuous-review perishable inventory systems. In the considered systems the goods at a distribution center used to fulfill unknown, variable demand deteriorate at a constant rate, and are replenished with delay from a remote supply source. We develop a new supply policy which incorporates the Smith predictor to counteract the adverse effects of delay. The proposed policy guarantees that the assigned storage space at the distribution center is never exceeded which means that the cost of emergency storage is eliminated. Moreover, we show that with appropriately chosen controller parameters all of the demand imposed at the distribution center is realized from the readily available resources.

Keywords: inventory control, perishable inventory systems, time-delay systems, Smith predictor.

1. INTRODUCTION

It follows from the extensive review papers documenting the research work in the past (Nahmias, 1982; Rafaat, 1991; Goyal and Giri, 2001; Ortega and Lin, 2004; Sarimveis et al., 2008; Karaesmen et al., 2008) that certain areas of inventory control are not sufficiently addressed at the formal design level. This concerns in particular a large and very important class of problems related to the management of perishable commodities (food, drugs, gasoline, etc.). The main difficulty in developing control schemes for perishable inventories stems from the necessity of conducting an exact analysis of product lifetimes. The design problem becomes cumbersome in the situation when the product demand is subject to significant uncertainty and inventories are replenished with nonnegligible delay, which frequently happens in modern supply chains. In such circumstances, in order to maintain high service level and at the same time keep stringent cost discipline, when placing an order it is necessary not only to account for the demand during procurement latency but also for the stock deterioration in that time.

Since the stock accumulation of perishables cannot be represented as a pure integrator, the effects of order procurement delay cannot be adequately accounted for by introducing the notion of work-in-progress or inventory position variables (constituting the sum of the on-hand and on-order goods), as has been done in a number of successful research works for nondecaying inventories, e.g. (Blanchini et al., 2000, Boccadoro et al., 2008). In contrast to our earlier results devoted exclusively to periodic-review inventory systems with nondeteriorating stock (Ignaciuk and Bartoszewicz, 2010a, b), in this work we analyze continuous-review systems with random lifetime of the stored goods. In order to solve the stability problems related to nonnegligible delay (see e.g. (Hoberg et al., 2007) for a discussion of the influence of delay on the dynamics of the traditional inventory systems), we propose to apply the Smith predictor (Smith, 1959). The designed control strategy is demonstrated to establish nonnegative and bounded ordering signal, which is a crucial requirement for the practical implementation of any replenishment rule. It is also shown that in the inventory system governed by the proposed policy the stock level never exceeds the assigned warehouse capacity, which means that the potential necessity for an expensive emergency storage outside the company premises is eliminated. At the same time, we demonstrate that the stock is never depleted, which implies full demand satisfaction from the readily available resources and 100% service level.

2. PROBLEM FORMULATION

We consider an inventory system where the goods at a distribution center used to fulfill the customers' (or retailers) demand are acquired with delay from a supply source. Such setting, illustrated in Fig. 1, is frequently encountered in production-inventory systems where a common point (distribution center), linked to a factory or an external, strategic supplier, is used to provide goods for another production stage or a distribution network. The task is to design a control strategy which, on one hand, will minimize the holding and shortage costs, and, on the other hand, will ensure smooth flow of goods despite unpredictable changes in market conditions.

3.1 System model

The imposed demand (the number of items requested from the distribution center) is modeled as an *a priori* unknown, bounded function of time $d(t)$, where t denotes time. We assume that demand can follow any statistical distribution as long as $0 \leq d(t) \leq d_{max}$, where d_{max} is a positive constant.

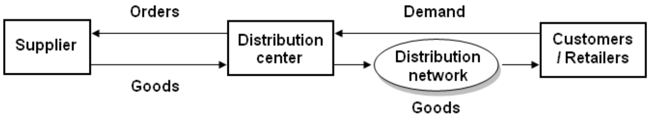

Fig. 1. Inventory system with a strategic supplier.

If there is a sufficient number of items at the distribution center to satisfy the imposed demand, then the actually met demand $h(t)$ (the number of items sold to customers or sent to retailers in the distribution network) will be equal to the requested one. Otherwise, the imposed demand is satisfied only from the arriving shipments, and the additional demand is lost (we assume that the sales are not backordered, and the excessive demand is equivalent to a missed business opportunity). Thus,

$$0 \le h(t) \le d(t) \le d_{\max}. \quad (1)$$

The on-hand stock used to fulfill the market demand deteriorates when kept in the distribution center warehouse at a constant rate σ, $0 \le \sigma < 1$. It is replenished with delay $L_p > 0$ from a remote supply source. Denoting the quantity ordered from the supplier at time t by $u(t)$, and the received shipment by $u_R(t)$, we have

$$u_R(t) = u(t - L_p). \quad (2)$$

Consequently, the stock balance equation can be written in the following way

$$\dot{y} = -\sigma y(t) + u_R(t) - h(t) = -\sigma y(t) + u(t - L_p) - h(t). \quad (3)$$

According to the stock balance equation, the on-hand stock decreases due to the realized sales represented by function $h(\cdot)$, and the decay characterized by factor σ. It is refilled from the goods acquired from the supplier $u_R(\cdot)$. For the sake of further analysis it is convenient to represent (3) in an integral form. We assume that initially the warehouse is empty, i.e. $y(0) = 0$, and the first orders are placed at $t = 0$, i.e. $u(t) = 0$ for $t < 0$. Solving (3) for $y(\cdot)$, we obtain (see the Appendix)

$$y(t) = \int_0^t e^{-\sigma(t-\tau)} u_R(\tau)\, d\tau - \int_0^t e^{-\sigma(t-\tau)} h(\tau)\, d\tau. \quad (4)$$

Since $u_R(t) = u(t - L_p)$ and $u(t < 0) = 0$, we can rewrite (4) in the following form

$$\begin{aligned} y(t) &= \int_0^t e^{-\sigma(t-\tau)} u(\tau - L_p)\, d\tau - \int_0^t e^{-\sigma(t-\tau)} h(\tau)\, d\tau \\ &= \int_0^{t-L_p} e^{-\sigma(t-L_p-\tau)} u(\tau)\, d\tau - \int_0^t e^{-\sigma(t-\tau)} h(\tau)\, d\tau. \end{aligned} \quad (5)$$

Note that in order to adequately model the stock accumulation of perishable goods, a saturating integrator needs to be applied, which makes the considered system nonlinear. However, if one can ensure that the control signal is nonnegative for arbitrary t, then by introducing the function representing the actually realized sales, $h(t) \le d(t)$, the stock dynamics can be reduced to linear equation (5). In the further part of the paper, we will design a control law which will be shown to satisfy the conditions $u(t) \ge 0$ and $h(t) = d(t)$. As a result, the inventory system will stay in the linear region of operation for the whole range of disturbance $0 \le d(t) \le d_{\max}$.

3.2 Transfer function representation

The linear part of the model of the considered inventory system with perishable goods can be described using transfer functions. The system block diagram is shown in Fig. 2. The saturating integrator in an internal loop represents the operation of accumulating the stock of perishables characterized by decay factor σ. The controller, with transfer function $G_C(s)$, is supposed to steer the on-hand stock level $y(t)$ towards the reference value y_{ref}, such that a high level of demand satisfaction is achieved.

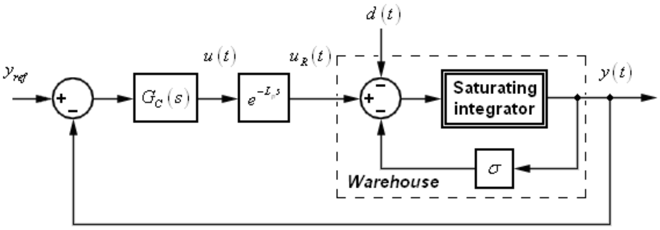

Fig. 2. System model.

3. PROPOSED CONTROL STRATEGY

The principal obstacle in providing efficient control in the considered class of systems is the latency in procuring orders. Indeed, each non-zero order placed at the supplier at instant t will appear at the distribution center with lead-time L_p at instant $t + L_p > t$ which may lead to oscillations, or even cause instability. In order to satisfactorily counteract the adverse effects of delay in the analyzed system with perishable goods, it is not sufficient to introduce inventory position variables (constituting the sum of on-hand stock and open orders), or the notion of work-in-progress, as it is usually done in the traditional inventory systems (Blanchini et al., 2000, Warburton, 2007; Boccadoro et al., 2008). This is due to the fact that the pure sum of open orders (or work-in-progress) does not account for the stock degradation within lead-time. To overcome the delay problem, in this work we propose to apply the Smith predictor (Smith, 1959), which proved a successful method of dead-time compensation in many engineering areas (Palmor, 1996). The basic idea behind the Smith predictor is to simulate the behavior of a remote plant inside the controller structure, thus eliminating the delay from the main feedback loop. The proposed control strategy, employing the Smith predictor for dead-time compensation is illustrated in Fig. 3.

The control structure consists of the primary plant controller $C(s)$ and the Smith predictor built on the linearized model of the plant $G^*(s) = 1/(s + \sigma)$. With the primary controller selected as the proportional control law $C(s) = K$, where K is a positive constant, we obtain the transfer function of the overall control structure $G_C(s)$,

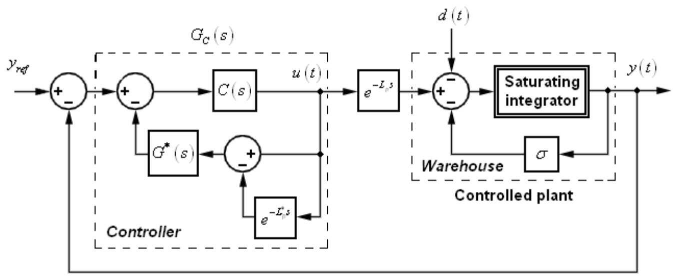

Fig. 3. Controller structure.

$$G_C(s) = \frac{C(s)}{1+C(s)\left[G^*(s)-e^{-L_p s}G^*(s)\right]} = \frac{K}{1+KG^*(s)\left(1-e^{-L_p s}\right)}. \quad (6)$$

In the linear region of operation the plant dynamics is fully represented by the transfer function $G(s) = 1/(s + \sigma)$. If the system parameters used by the controller match those of the true object, i.e. when $e^{-L_p^* s}G^*(s) = e^{-L_p s}G(s)$, then we can write the closed-loop transfer functions:

a) with respect to the reference input $Y_{ref}(s) = y_{ref} / s$

$$\frac{Y(s)}{Y_{ref}(s)} = \frac{K}{s+\sigma+K}e^{-L_p s}, \quad (7)$$

b) with respect to the disturbance $D(s) = \mathcal{L}(d(t))$,

$$\frac{Y(s)}{D(s)} = -\frac{1}{s+\sigma} + \frac{K}{s+\sigma+K}e^{-L_p s}. \quad (8)$$

It is clear from (7) and (8) that the term related to delay is eliminated from the characteristic equation (the denominator of the closed-loop transfer function). Consequently, since $K > 0$ and $\sigma \geq 0$, the closed-loop system under nominal operating conditions is stable for arbitrary lead-time and any bounded disturbance.

4. PROPERTIES OF THE PROPOSED STRATEGY

Before we state the properties of the proposed inventory policy (6), it is convenient to present it in time domain. We assume that the controller has the exact knowledge of the system parameters. Taking into account the initial conditions, we can write the control law in time domain by direct inspection of the block diagram shown in Fig. 3 in the following form

$$\begin{aligned} u(t) &= K\left[y_{ref} - y(t)\right] \\ &\quad - K\left[\int_0^t e^{-\sigma(t-\tau)}u(\tau)d\tau - \int_0^{t-L_p} e^{-\sigma(t-L_p-\tau)}u(\tau)d\tau\right]. \end{aligned} \quad (9)$$

This control law can be interpreted as to generate orders in proportion to the difference between the current on-hand stock and its reference value $K(y_{ref} - y(t))$ decreased by the amount of open orders quantified by the rate of deterioration within the last lead-time (the terms in the square brackets).

The properties of the proposed control strategy will be given in three Theorems, and strictly proved. The first theorem shows that the ordering signal generated by the controller is always nonnegative and bounded, which is a crucial prerequisite for the implementation of any cost-efficient inventory management policy. The second proposition specifies the upper bound of the on-hand stock, which constitutes the smallest warehouse capacity required to store all the incoming shipments. Finally, the third theorem shows how to select the stock reference value in order to guarantee that all of the imposed demand will be fulfilled from the readily available resources at the distribution center thus ensuring the maximum service level.

Theorem 1: The ordering signal generated by controller (9) applied to system (3) satisfies the following inequalities

$$K\frac{\sigma y_{ref}}{\sigma+K} \leq u(t) \leq Ky_{ref}. \quad (10)$$

Moreover, there exists a time instant t_0 such that for any $t \geq t_0$

$$u(t) \leq K\frac{\sigma y_{ref} + d_{\max}}{\sigma+K}. \quad (11)$$

Proof: Substituting (5) into (9) we get

$$u(t) = K\left[y_{ref} - \int_0^t e^{-\sigma(t-\tau)}u(\tau)d\tau + \int_0^t e^{-\sigma(t-\tau)}h(\tau)d\tau\right]. \quad (12)$$

Consequently,

$$\begin{aligned} \dot{u} &= -K\frac{d}{dt}\left\{e^{-\sigma t}\int_0^t e^{\sigma\tau}\left[u(\tau)-h(\tau)\right]d\tau\right\} \\ &= K\left\{\sigma e^{-\sigma t}\int_0^t e^{\sigma\tau}\left[u(\tau)-h(\tau)\right]d\tau - e^{-\sigma t}e^{\sigma t}\left[u(t)-h(t)\right]\right\} \quad (13) \\ &= K\left\{\sigma\int_0^t e^{-\sigma(t-\tau)}\left[u(\tau)-h(\tau)\right]d\tau - \left[u(t)-h(t)\right]\right\}. \end{aligned}$$

It follows from (12) that

$$\sigma K\int_0^t e^{-\sigma(t-\tau)}\left[u(\tau)-h(\tau)\right]d\tau = \sigma\left[Ky_{ref} - u(t)\right]. \quad (14)$$

Hence, we can rewrite (13) as

$$\begin{aligned} \dot{u} &= \sigma\left[Ky_{ref} - u(t)\right] - K\left[u(t)-h(t)\right] \\ &= \sigma Ky_{ref} - (\sigma+K)u(t) + Kh(t). \end{aligned} \quad (15)$$

Investigating $\dot{u} = 0$ we get

$$u(t) = K\frac{\sigma y_{ref} + h(t)}{\sigma+K}. \quad (16)$$

According to constraint (1) the minimum satisfied demand equals zero. At the initial time $u(0) = Ky_{ref} > 0$. Therefore, since $0 \leq \sigma < 1$ and $h(\cdot) \geq 0$, we get from (16) that $u(\cdot)$ decreases as long as it is bigger than $K[\sigma y_{ref} + h(\cdot)]/(\sigma + K)$, and it never falls below $K\sigma y_{ref}/(\sigma + K)$. Moreover, there exists a time instant t_0 when $u(\cdot)$ reaches the level of $K[\sigma y_{ref} + d_{\max}]/(\sigma + K)$ for the first time. Since $h(\cdot) \le d_{\max}$, we get from (16) that for all $t \ge t_0$

$$u(t) \le K\frac{\sigma y_{ref} + d_{\max}}{\sigma + K}.$$

This conclusion ends the proof. □

Theorem 2: If policy (9) is applied to system (3), then the on-hand stock at the distribution center never exceeds the level of y_{ref} for $\sigma = 0$ and

$$y_{\max} = \frac{K}{\sigma + K}\left[y_{ref} + \frac{d_{\max}}{\sigma}\left(1 - e^{-\sigma L_p}\right)\right] \quad \text{for } \sigma > 0. \tag{17}$$

Proof: Applying (12) to the stock balance equation (3) we get

$$\begin{aligned}\dot{y} &= -\sigma y(t) + Ky_{ref} \\ &\quad - K\left[\int_0^{t-L_p} e^{-\sigma(t-L_p-\tau)}u(\tau)d\tau - \int_0^t e^{-\sigma(t-\tau)}h(\tau)d\tau\right] \\ &\quad + K\int_0^{t-L_p} e^{-\sigma(t-L_p-\tau)}h(\tau)d\tau - K\int_0^{t-L_p} e^{-\sigma(t-\tau)}h(\tau)d\tau \\ &\quad - K\int_{t-L_p}^t e^{-\sigma(t-\tau)}h(\tau)d\tau - h(t).\end{aligned} \tag{18}$$

Using (5) we can notice that the term in the square brackets in (18) actually equals $y(t)$. Consequently, we have

$$\begin{aligned}\dot{y} &= Ky_{ref} - (\sigma + K)y(t) - h(t) \\ &\quad + K\left(e^{\sigma L_p} - 1\right)\int_0^{t-L_p} e^{-\sigma(t-\tau)}h(\tau)d\tau - K\int_{t-L_p}^t e^{-\sigma(t-\tau)}h(\tau)d\tau.\end{aligned} \tag{19}$$

Closer investigation of $\dot{y} = 0$ leads to

$$\begin{aligned}y(t) &= \frac{Ky_{ref}}{\sigma + K} + \frac{K}{\sigma + K}\left(e^{\sigma L_p} - 1\right)\int_0^{t-L_p} e^{-\sigma(t-\tau)}h(\tau)d\tau \\ &\quad - \frac{K}{\sigma + K}\left[\int_{t-L_p}^t e^{-\sigma(t-\tau)}h(\tau)d\tau + \frac{h(t)}{K}\right].\end{aligned} \tag{20}$$

It follows from (20) that since $K > 0$, $\sigma \ge 0$, and $h(\cdot) \ge 0$, the biggest value of $y(\cdot)$ is expected when $h(\tau) = d_{\max}$ for $\tau \le t - L_p$ and $h(\tau) = 0$ in the interval $(t - L_p, t]$. We get immediately from (20) that for $\sigma = 0$ (the case of nondeteriorating stock) $y(t) \le y_{ref}$. Evaluating the first integral in (20) for $\sigma > 0$ we obtain

$$\begin{aligned}&\int_0^{t-L_p} e^{-\sigma(t-\tau)}h(\tau)d\tau \le d_{\max}\int_0^{t-L_p} e^{-\sigma(t-\tau)}d\tau \\ &= d_{\max}e^{-\sigma t}\int_0^{t-L_p} e^{\sigma\tau}d\tau = d_{\max}\frac{e^{-\sigma t}}{\sigma}\left(e^{\sigma\tau}\right)\Big|_0^{t-L_p} \\ &= d_{\max}\frac{e^{-\sigma t}}{\sigma}\left[e^{\sigma(t-L_p)} - 1\right] = \frac{d_{\max}}{\sigma}\left[e^{-\sigma L_p} - e^{-\sigma t}\right] \le \frac{d_{\max}}{\sigma}e^{-\sigma L_p}.\end{aligned} \tag{21}$$

Consequently, applying (21) to (20), we arrive at

$$y(t) \le \frac{K}{\sigma + K}\left[y_{ref} + \frac{d_{\max}}{\sigma}\left(1 - e^{-\sigma L_p}\right)\right]. \tag{22}$$

This ends the proof. □

It follows from Theorem 2 that if the warehouse of size $y_{\max}$ specified by (17) is assigned at the distribution center, then all the incoming shipments can be stored locally, and any cost associated with emergency storage is eliminated. Apart from the efficient warehouse space management, a successful inventory control strategy in modern supply chain is expected to achieve a high level of demand satisfaction. The proposition formulated below shows how the reference stock level should be selected so that $y(t) > 0$, which implies that all of the demand imposed on the distribution center is satisfied from the readily available resources.

Theorem 3: If policy (9) is applied to system (3), and the reference stock level is selected as

$$y_{ref} > d_{\max}\left(L_p + 1/K\right) \quad \text{for } \sigma = 0, \tag{23}$$

$$y_{ref} > d_{\max}\left[\left(1 - e^{-\sigma L_p}\right)/\sigma + 1/K\right] \quad \text{for } \sigma > 0, \tag{24}$$

then the on-hand stock level at the distribution center is strictly positive for any $t > L_p$.

Proof: Note that $e^{\sigma L_p} - 1 > 0$. Hence, considering (1) and (20), we can expect the smallest on-hand stock level in the circumstances when $h(\tau) = 0$ for $\tau \le t - L_p$ and $h(\tau) = d_{\max}$ for τ belonging to the interval $(t - L_p, t]$. The warehouse is empty for $t \le L_p$. In the case of the system with nondeteriorating stock ($\sigma = 0$) we get immediately from (20)

$$y(t) \ge y_{ref} - d_{\max}\left(L_p + 1/K\right). \tag{25}$$

Thus, based on assumption (23) we have $y(t) > 0$ for $\sigma = 0$. Evaluating the second integral in (20) for $t > L_p$ in the case when $h(t) = d_{\max}$ and $\sigma > 0$, we obtain

$$\begin{aligned}&\int_{t-L_p}^t e^{-\sigma(t-\tau)}h(\tau)d\tau \le d_{\max}\int_{t-L_p}^t e^{-\sigma(t-\tau)}d\tau \\ &\quad = d_{\max}e^{-\sigma t}\int_{t-L_p}^t e^{\sigma\tau}d\tau = d_{\max}\frac{e^{-\sigma t}}{\sigma}e^{\sigma\tau}\Big|_{t-L_p}^t \\ &\quad = d_{\max}\frac{e^{-\sigma t}}{\sigma}\left[e^{\sigma t} - e^{\sigma(t-L_p)}\right] = \frac{d_{\max}}{\sigma}\left[1 - e^{-\sigma L_p}\right].\end{aligned} \tag{26}$$

Applying (26) to (20), we get the on-hand stock level $y(\cdot)$ at the instant when it is minimal

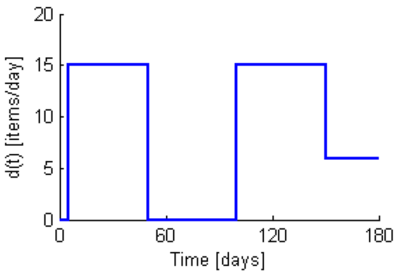

Fig. 4. Market demand.

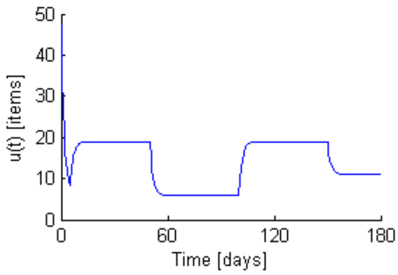

Fig. 5. Orders placed at the supplier.

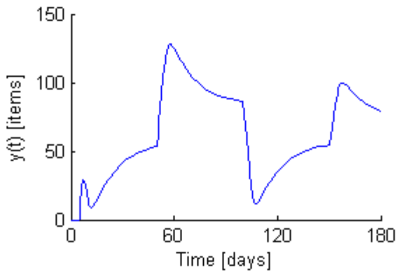

Fig. 6. On-hand stock level.

$$y(t) \geq \frac{K}{\sigma + K}\left[y_{ref} - \frac{d_{\max}}{\sigma}\left(1 - e^{-\sigma L_p}\right) - \frac{d_{\max}}{K}\right]. \tag{27}$$

If the reference stock level is adjusted according to (24), then using (27) one may conclude that

$$y(t) \geq \frac{K}{\sigma + K}\left\{ y_{ref} - d_{\max}\left[\left(1 - e^{-\sigma L_p}\right)/\sigma + 1/K\right]\right\} > 0. \tag{28}$$

This completes the proof. □

Remark: It follows from Theorem 1 that the controller generates ordering signal that is always nonnegative and bounded, which makes the considered system positive. In addition, if assumptions of Theorem 3 are fulfilled, then $h(t) = d(t)$, and the plant remains in the linear region for arbitrary demand satisfying condition (1). Considering the responses with respect to the reference input (7) and with respect to the disturbance (8) we get the overall system transfer function

$$Y(s) = \frac{Ke^{-L_p s}}{s + \sigma + K}\frac{y_{ref}}{s} - \left(\frac{1}{s+\sigma} - \frac{K}{s + \sigma + K}e^{-L_p s}\right)D(s). \tag{29}$$

Consequently, applying the final value theorem we get the steady-state stock level y_{ss} (the stock level in the presence of the steady-state demand $d_{ss} > 0$)

$$y_{ss} = \lim_{s\to 0} sY(s) = \begin{cases} y_{ref} - \dfrac{1 + KL_p}{K} d_{ss} & \text{for } \sigma = 0, \\ \dfrac{Ky_{ref}}{\sigma + K} - \left(\dfrac{1}{\sigma} - \dfrac{K}{\sigma + K}\right) d_{ss} & \text{for } \sigma > 0. \end{cases} \tag{30}$$

Equation (30) indicates that a finite steady-state error will be present at the output when a proportional control law is chosen as the primary controller $C(s)$ in (6). Typically in engineering systems, this error would need to be reduced (or eliminated), for instance by introducing a proportional-integral controller in place of the proportional one. Also a feed-forward term could be applied to compensate the effects of disturbance. However, in the considered application, y_{ref} can be assigned an arbitrary value and any steady-state error can be tolerated. What is important from the practical point of view when studying inventory control problems is the size of the required storage space and demand utilization. Theorems 2 and 3 show precisely how much storage space should be provided to accommodate all the incoming shipments (relation (17)), and how to select y_{ref} to guarantee that all the sales are realized from the readily available resources (inequalities (23) and (24)).

4. NUMERICAL EXAMPLE

The properties of the designed policy (9) are verified in simulations conducted for the model of perishable inventory system described in Section 2. The system parameters are set in the following way: lead-time $L_p = 5$ days, inventory decay factor $\sigma = 0.07$ day^{-1}, and the maximum daily demand at the distribution center $d_{\max} = 15$ items/day. The actual demand follows the pattern illustrated in Fig. 4, which reflects abrupt seasonal changes in a half-year trend. The controller gain is adjusted to $K = 0.5$. In order to ensure full demand satisfaction, the stock reference level is set according to the guidelines of Theorem 3 as $y_{ref} = 95 > 93$ items. This results in the required storage space calculated according to (17) $y_{\max} = 139$ items.

The orders generated by controller (9) in response to the demand pattern from Fig. 4 are shown in Fig. 5, and the resultant on-hand stock in Fig. 6. We can see from the graphs depicted in Fig. 5 that the proposed controller quickly responds to the sudden changes in the demand trend without oscillations or overshoots in the ordering signal. For $t \geq t_0 = 2$ days the order quantity remains in the interval [5.83, 19.01 items/day], precisely as dictated by Theorem 1. The knowledge about the range of order changes helps in establishing long-term relationship between the distribution center and the supplier, and facilitates capacity planning down the supply chain (at the supplier and its subcontractors). We can see from Fig. 6 that the stock level does not increase beyond $y_{\max} = 139$ items, which means that the assigned warehouse capacity is sufficient to store the goods at the distribution center at all times. Moreover, the on-hand stock never falls to zero after the initial phase which implies full demand satisfaction and 100% service level.

5. CONCLUSIONS

In this work we designed a new inventory management policy for continuous-time inventory systems with perishable goods. The proposed policy employs the Smith predictor for compensating the adverse effects of order procurement delay. As a result, the system stability is guaranteed for arbitrary delay and any bounded demand pattern. The ordering signal generated by the designed policy smoothly adapts to the demand changes, and thus it is easy to follow by the supplier.

The ordering signal is proved to remain finite and always nonnegative, which is crucial for the practical implementation of any inventory management scheme. It is also demonstrated in the paper that the stock level resulting from the application of the proposed policy does not increase beyond the precisely determined warehouse capacity, which eliminates the need for costly emergency storage and facilitates capacity planning at the distribution center. Finally, it is shown how to select controller parameters to achieve full satisfaction of the unknown market demand.

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REFERENCES

Blanchini, F., Pesenti, R, Rinaldi, F., and Ukovich, W. (2003). Feedback control of production-distribution systems with unknown demand and delays. *IEEE Transactions on Robotics and Automation*, *16*(3), 313–317.

Boccadoro, M., Martinelli, F., and Valigi, P. (2008). Supply chain management by H-infinity control," *IEEE Transactions on Automation Science and Engineering*, *5*(4), 703–707.

Goyal, S.K., and Giri, B.C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research*, *134*(1), 1–16.

Hoberg, K., Bradley, J.R., and Thonemann, U.W. (2007). Analyzing the effect of the inventory policy on order and inventory variability with linear control theory. *European Journal of Operational Research*, *176*(3), 1620–1642.

Ignaciuk, P., and Bartoszewicz, A. (2010). LQ optimal sliding mode supply policy for periodic review inventory systems," *IEEE Transactions on Automatic Control*, *55*(1), 269–274.

Ignaciuk, P., and Bartoszewicz, A. (2010). LQ optimal and reaching law based sliding modes for inventory management systems," *International Journal of Systems Science*, *41* (*in press*).

Karaesmen, I., Scheller-Wolf, A., and Deniz, B. (2008). Managing perishable and aging inventories: review and future research directions. In: Kempf, K., Keskinocak, P., and Uzsoy, R. (eds.). *Handbook of production planning*. Dordrecht: Kluwer.

Nahmias, S. (1982). Perishable inventory theory: a review. *Operations Research*, *30*(4), 680–708.

Ortega, M., and Lin, L. (2004). Control theory applications to the production-inventory problem: a review. *International Journal of Production Research*, *42*(11), 2303–2322.

Palmor, Z.J. (1996). Time-delay compensation – Smith predictor and its modifications. In Levine, W.S. (ed.) The Control Handbook, CRC Press, 224–237.

Rafaat, F. (1991). Survey of literature on continuously deteriorating inventory models. *Journal of the Operational Research Society*, *42*(1), 27–37.

Sarimveis, H., Patrinos, P., Tarantilis, C.D., and Kiranoudis, C.T. (2008). Dynamic modeling and control of supply chain systems: a review. *Computers & Operations Research*, *35*(11), 3530–3561.

Smith, O.J.C. (1959). A controller to overcome dead time. *ISA Journal*, *6*(2), 28–33.

Warburton, R.D.H. (2007). An optimal, potentially automatable ordering policy. *International Journal of Production Economics*, *107*(2), 483–495.

APPENDIX

We solve differential equation (3) with the initial conditions: $y(0) = 0$, and $u_R(t) = u(t - L_p) = 0$ for $t < L_p$. First we consider the homogeneous equation

$$\dot{y} + \sigma y(t) = 0, \tag{31}$$

which leads to

$$y(t) = y(0)e^{-\sigma t}. \tag{32}$$

In order to determine the nonhomogeneous solution we assume $y(t)$ in the following form

$$y(t) = q(t)e^{-\sigma t}, \tag{33}$$

where $q(t)$ is a function differentiable with respect to time. Differentiating both sides of (33) we obtain

$$\dot{y} = \dot{q}e^{-\sigma t} - \sigma q(t)e^{-\sigma t}. \tag{34}$$

Substituting (33) and (34) into (3), we get

$$\dot{q}e^{-\sigma t} = u_R(t) - h(t). \tag{35}$$

Solving (35) for $q(t)$ yields

$$q(t) = \int_0^t e^{\sigma\tau}\left[u_R(t) - h(t)\right]d\tau + C, \tag{36}$$

where C is the constant of integration. Substituting (36) into (33), we arrive at

$$\begin{aligned} y(t) &= \left\{\int_0^t e^{\sigma\tau}\left[u_R(t) - h(t)\right]d\tau + C\right\}e^{-\sigma t} \\ &= Ce^{-\sigma t} + \int_0^t e^{-\sigma(t-\tau)}\left[u_R(t) - h(t)\right]d\tau. \end{aligned} \tag{37}$$

Applying the initial condition $y(0) = 0$, we have $C = 0$, and

$$y(t) = \int_0^t e^{-\sigma(t-\tau)}\left[u_R(t) - h(t)\right]d\tau. \tag{38}$$